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1990 J. Phys.: Condens. Matter 2 8945

(http://iopscience.iop.org/0953-8984/2/45/008)

J. Phys.: Condens. Matter **2** (1990) 8945–8952. Printed in the UK

A theory of wall mobility in weakly damped insulating ferromagnets

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Received 21 January 1990, in final form 9 May 1990

Abstract. The classical calculation of the Bloch wall mobility, based on the Landau–Lifschitz–Gilbert equation, leads to a result which is 1000 times larger than the experimental value found for very pure yttrium iron garnet. We criticize this approach and propose a novel mechanism. It is argued that the non-zero transverse magnetization of the wall induces a magnetic back-flow around the wall, and hence a drag force proportional to the velocity. A correct order of magnitude of the mobility is obtained if one *assumes* that the intrinsic relaxation times of the medium are very different for high frequencies (where resonance experiments are performed) and low frequencies, relevant to the wall motion.

1. Introduction

There exists quite a vast body of experimental results on the domain wall mobility in ferromagnets, resulting from many years of investigation. In *conducting* ferromagnets, the domain wall motion is damped by eddy current losses and hence its mobility can be expressed in terms of the conductivity of the material (Williams *et al* 1950) and good agreement with experimental data is achieved. In *insulating* ferromagnets, such as ferrites, spinels or garnets, coupling between the spin degrees of freedom and all the other degrees of freedom (the thermal bath) is much more indirect, in particular if the material is (nearly) impurity and defect free. A possible quantitative measure of this coupling is provided by the width ΔH of the ferromagnetic resonance line which gives the relaxation time $\tau^{-1} = \gamma \Delta H$ for uniform Larmor precession of the spins around the total magnetic field, at the frequency $\omega_r = \gamma H$ (γ is the gyromagnetic ratio). With this information, the classical way of estimating the mobility of a domain wall is the following: the evolution of each spin $\boldsymbol{S}$ is assumed to be described by the Landau–Lifschitz–Gilbert (LLG) equation

$$\partial \boldsymbol{S}/\partial t = \gamma \boldsymbol{S} \times \boldsymbol{H} - (\gamma \alpha \boldsymbol{S}/S) \times (\boldsymbol{S} \times \boldsymbol{H})$$

where the first term describes the Larmor precession and the second term is a phenomenological damping term which conserves $|S|^2$. $\boldsymbol{H}$ is the total field acting on $\boldsymbol{S}$, including in particular the exchange field and the anisotropy field. For zero external field, the stationary solution of this equation with $\boldsymbol{S} = \boldsymbol{S}(x)$ and $\boldsymbol{S}(\pm\infty) = \pm S\boldsymbol{z}$ leads to a domain wall magnetization profile of width δ. For non-zero external field H_0, one can find a travelling solution $S = S(x - Vt)$ (*no Bloch lines*) with

$$V = \delta\gamma H_0/\alpha.$$

Assuming α constant, independent of the total field H, one obtains a linear relation between V and H_0 characterized by a mobility $\mu = \delta\gamma/\alpha$. Solving LLG in the ferromagnetic resonance situation leads to $\alpha = \Delta H/2H$, which allows one to compare linewidth measurements with domain wall mobility. Experimentally, the situation is as follows: if damping is 'high' ($\alpha \geqslant 0.1$), due to the presence of strongly coupled impurities, this relation is qualitatively correct, i.e. $\alpha_\mu/\alpha_{\Delta H} \simeq 1$. If the damping is weak, in particular in pure single-crystal yttrium iron garnet (YIG), many experiments (Hagedorn and Gyorgy 1961, Harper and Teale 1969, Vella-Colleiro *et al* 1972, Guyot *et al* 1988) (see in particular Teale (1980) where a clear discussion of the problem is given) have shown that $\alpha_\mu/\alpha_{\Delta H} \simeq 1000$! (The mobility is of the order of $30\ \mathrm{m\,s^{-1}\,Oe^{-1}}$.) This obviously shows that a better theory of domain wall mobility is lacking, since experiments on YIG are very 'clean' and well controlled. The following ways to remedy this may be considered.

(i) It is well known that the presence of Bloch lines drastically reduces the mobility. For a domain wall with a typical distance between Bloch lines equal to ξ, the reduction factor is (see, e.g., Eschenfelder 1982, and references therein) about $\alpha^2(\xi/\delta_B)$, where δ_B is the width of a Bloch line. In order to obtain the correct mobility, the distance between Bloch lines should thus be of the order of 10 cm: hence, the number of Bloch lines present should be less than 1! Furthermore, the transverse velocity of Bloch lines can be shown to be $1/\alpha$ times the velocity of the wall; the very few Bloch lines present would thus quickly reach the edge of the sample. This would hence lead to a very intermittent overall wall motion, with 'fast' periods occurring during a time of the order of the nucleation time of a Bloch line, separated by short 'bursts' of slower velocity.

We rather believe that the Bloch lines are quite numerous to reduce the magnetostatic energy of the wall, but also *quite strongly interacting* so as to make the network of Bloch lines solid or glassy. We thus suggest that Bloch lines are essentially immobile and do not participate directly in the energy dissipation†.

(ii) α may change appreciably from low frequencies (where mobility measurements are usually performed) to high frequencies (ferromagnetic resonance at around 10 GHz). This possibility was also ruled out by Teale, who was able to drive the wall sufficiently fast that the frequencies involved in its motion (about V/δ) were of the order of 1 GHz, with no sign that the mobility increases rapidly. From a theoretical point of view, however, there is no microscopic justification for a constant α‡ (τ can indeed be calculated in some cases (Haas and Callen 1963) and is generally not found to behave as H^{-1}, as would require a constant α). It would be in fact quite natural to think in terms of a relaxation time, i.e. to write the damping part of the LLG equation as

$$(\tau^{-1}/SH)\boldsymbol{S} \times (\boldsymbol{S} \times \boldsymbol{H})$$

and to suppose rather that, for weak fields, τ approaches a constant. This would not lead to a linear relation between V and H_0. Hence, in our view, the hypothesis of a 'fluid'

† They are, however, important in determining the magnetic field created by the wall—see below.

‡ See, however, the very recent paper of Plefka (1990), where a Langevin theory of the LLG equation is constructed.

damping (energy dissipation proportional to $\alpha\dot{S}^2$), and hence the very existence of a domain wall mobility in this approach is quite *ad hoc*.

(iii) The LLG equation is phenomenological in nature; it is based on the fact that the magnitude of the spin is fixed by the competition between exchange energy and entropy, which are characterized by energies much larger than those involved in the dynamical processes studied here. Hence $|S|$ is essentially locked onto its thermal equilibrium value. The damping term must thus be orthogonal to S and must be zero for $S \parallel H$. One could generalize LLG, in the spirit of phenomenological approaches, and write†

$$\frac{\partial S(r,t)}{\partial t} = \gamma S(r,t) \times H(r,t) - \frac{\tau^{-1}}{SH} S(r,t) \times \left(\int_{V,t} \mathrm{d}^3 r' \, \mathrm{d}t' \, S(r',t') K(r-r', t-t') \times H \right). \tag{1}$$

It is obvious that the LLG equation is recovered for $K(u, v) \sim \delta(u)\delta(v)$. If one takes a kernel local in time and decaying fast in space, one will add to the LLG damping term an 'inhomogeneous damping' contribution of the form

$$(D_{\mathrm{m}}/SH) S \times (\nabla^2 S \times H) \tag{2}$$

with D_{m} having the dimensions of a diffusion constant. For the magnetization profile inside a domain wall, one has $|\nabla^2 S| \simeq S\delta^{-2}$, and hence the effective inverse relaxation time is (see also Bar'yakthar (1984)) $\tau^{-1} + D_{\mathrm{m}}\delta^{-2}$. Taking $D_{\mathrm{m}} \simeq Ja^2$ with J a characteristic exchange frequency, one finds that (with $\delta = 100a$) $D_{\mathrm{m}}\delta^{-2} \simeq 1$ GHz (leading indeed to the correct order of magnitude for the mobility), while the homogeneous relaxation time is (in YIG) about 1 MHz. This shows that 'inhomogeneous damping' could be dominant in situations where magnetization varies quickly, as in a domain wall. However, one still should assume an H-dependent D_{m} = constant H in order to obtain a linear relation between V and H; furthermore it is difficult to give a serious order of magnitude of what appears as a new phenomenological parameter. Note that this approach predicts a mobility diverging as δ^3 as the Curie temperature is approached.

The possibility that we shall explore in this paper is that the mechanism slowing down the domain wall motion and that governing the ferromagnetic resonance have a *fundamentally different origin* when the intrinsic damping is small‡. A very important feature of the Bloch domain walls is that they have a net magnetization in the plane of the wall, which creates a demagnetizing field and polarizes its surroundings. Bloch lines break up the wall into 'domains' to reduce the magnetostatic energy and thus determine the precise structure of the magnetic field around the wall.

When the wall is moving, the polarization cloud is asymmetrical and imposes a viscous drag (proportional to the velocity for small V); the 'charge' is attracted by its image (the 'Narcissus effect'). The back-flow mechanism that we propose for domain wall mobility is thus exactly the same as the slowing down of a charged particle in a

† After this paper was accepted, our attention was brought to the work of Bar'yakthar (1984) (see also Dorman and Sokolev (1988)), where this idea has already been explicitly investigated. In particular, the wall mobility is shown to be reduced by a factor $1 + \tau D_{\mathrm{m}}/3\delta^2$ (see equation (2) below).

‡ This is also the point of view taken by Thiele and Asselin (1984) who propose that spin waves *localized in the Bloch wall* are generated when the wall sweeps an impurity. This leads to a specific energy dissipation, which contributes both to the coercivity and to the wall mobility. It is, however, difficult to extract from these calculations (which are only briefly sketched by Thiele and Asselin) an actual order of magnitude of the mobility of very pure samples, where this mechanism is *a priori* not expected to be very efficient.

dielectric medium (see e.g., Landau and Lifschitz 1985) or the mobility of grain boundaries interacting with solute atoms. It is similar in spirit to the qualitative idea of a 'magnetic wake' left behind by the wall expressed by Hagedorn and Gyorgy (1961). We obtain, using known information about the samples considered, an estimate of the mobility which compares quite favourably with experiments. This approach also suggests that the mobility measured in AC experiments should increase with increasing frequency and that the mobility should be much higher for Néel walls in films, where the net magnetization is perpendicular to the wall and which does not leave a magnetic trace. This is indeed the case in YIG films, where the mobility is quite well estimated by the LLG equations (de Leeuw *et al* 1980, p 774).

2. Model and theoretical results

The problem can be cast in a quite general form. Assume a 'particle'—which can be an electric charge, a domain wall, a grain boundary, etc—immersed in a polarizable medium, characterized by an internal variable (electric or magnetic polarization, concentration, stress, etc) denoted by $\varphi(\boldsymbol{r}, t)$. The particle creates a polarizing field $h(\boldsymbol{r}, t) = \kappa\delta(\boldsymbol{r} - \boldsymbol{r}(t))$, to which the medium responds as

$$\varphi(\boldsymbol{q}, \omega) = \chi(\boldsymbol{q}, \omega)h(\boldsymbol{q}, \omega) \tag{3}$$

where Fourier transforms have been introduced.

Assuming that the particle has a uniform motion $\boldsymbol{r} = \boldsymbol{V}t$, it is easy to show that the viscous drag imposed by the polarization cloud is given by (see, e.g. Landau and Lifschitz (1985) and Pines and Nozières (1966) for similar calculations):

$$F_{\mathrm{drag}} = \frac{\kappa^2\chi_0}{2}\Big(\sum_{\mathrm{residues}+} q\chi(q, Vq) - \sum_{\mathrm{residues}-} q\chi(q, Vq)\Big) \tag{4}$$

where $\chi_0 \equiv \chi(0, 0)$ and $\Sigma_{\mathrm{residues}\pm}$ denotes the sum over all residues at the poles in the upper and lower half-q-planes, respectively.

In the case of a medium in which φ diffuses with a diffusion constant D and is damped at a rate τ^{-1}, one has

$$\chi(q, \omega) = \chi_0[1/(Dq^2\tau + \mathrm{i}\omega\tau + 1)]. \tag{5}$$

In the limit of small velocities ($V < V^* = 2\sqrt{D\tau^{-1}}$), one finds that†

$$F_{\mathrm{drag}} = (\kappa^2\chi_0/4\sqrt{D^3\tau})V \equiv V/\mu_{\mathrm{drag}}. \tag{6}$$

The mobility of the object is then obtained by writing $V = \mu_{\mathrm{intr}}\,(F - F_{\mathrm{drag}})$ where μ_{intr} takes into account other sources of dissipation. Hence $\mu^{-1} = \mu_{\mathrm{intr}}^{-1} + \mu_{\mathrm{drag}}^{-1} \simeq \mu_{\mathrm{drag}}^{-1}$ for small intrinsic dissipation.

It is in fact helpful for physical applications to write the expression of the drag force for a more general law of motion $x(t)$, without assuming a constant velocity V. This reads

$$F_{\mathrm{drag}}(t) = \kappa^2\chi_0\int_{-\infty}^{t}\frac{\mathrm{d}t'}{\sqrt{4\pi D(t-t')}}\frac{1}{\tau}\exp\Big(-\frac{t-t'}{\tau}\Big)\exp\Big(-\frac{[x(t)-x(t')]^2}{4D(t-t')}\Big)\frac{x(t')-x(t)}{2D(t-t')} \tag{7}$$

† A similar expression can be found in the work of de Gennes (1988), where the influence of a polarizable solvent on the conductivity of stretched polymers is studied.

in which each term is quite easy to interpret physically. Equation (7) reproduces the above result (6) for constant velocities. This expression will allow us to *estimate* the order of magnitude of the drag force in more complex situations (see below).

The problem that we consider here is slightly more complicated since *spin waves* can propagate in the medium. The susceptibility of polarized ferrimagnets is given by

$$\chi(q, \omega) = \gamma 4\pi M_s[H_a(q) + (\mathrm{i}\omega + \tau^{-1})\tau^{-1}/H_a(q)]/[H_a^2(q) + \tau^{-2} - \omega^2 + 2\mathrm{i}\omega\tau^{-1}]. \tag{8}$$

M_s is the saturation magnetization, and $H_a(q) = H_a^0 + Dq^2$ where H_a^0 is the anisotropy field (multiplied by γ). $\tau = \tau(q, \omega)$ is the intrinsic relaxation time of the medium, which may depend on both the wavevector and the frequency (e.g. the linewidth of the ferromagnetic resonance line is obtained with $q = 0$, $\omega = \gamma H$). We shall define the low-frequency relaxation time as $\tau_0 \equiv (\sqrt{\Omega/D}, 0)$ with $\Omega = \sqrt{\tau^{-2} + H_a^{02}}$; the analysis of equation (5) with $\chi(q, \omega)$ given by (8) indeed reveals that Ω is the characteristic frequency separating the low- and high-frequency regimes.

For small velocities $V < V^* = 2\sqrt{D\Omega}$, equation (5) leads to the following results.

(i) If $\tau_0^{-1} \gg H_a^0$, then the mobility determined by the magnetic drag is equal to the value determined above in equation (6) (up to a numerical factor).

(ii) If on the contrary $\tau_0^{-1} \ll H_a^0$, then the mobility is *enhanced* by a factor $\sqrt{2\tau_0 H_a^0}$.

The 'coupling' constant κ measuring the field created by the wall is of the order of $\delta 4\pi M_s$. As the static susceptibility is $M_s/H_a \gg 1$ for most ferrites, the local response to this field is $H = 4\pi M_s$ (saturation) and not $\chi_0 4\pi M_s$. Hence, the drag force in this strong-coupling case is a factor χ_0 smaller than that given above. Moreover, the above calculation only makes sense if the diffusion length $\sqrt{D\tau}$ is small compared with the screening length ξ (due to Bloch lines within the wall). In the opposite case, the integral of equation (7) must be cut at $\tau_{max} = \xi^2/D$.

The 'diffusion constant' is related through the spin-wave stiffness and hence the Curie temperature through

$$D \sim Ja^2 \sim (T_c/\hbar)a^2 \tag{9}$$

(a is the lattice spacing). Note that usually the domain wall mobility is rather defined as $V = \mu H_0$ where H_0 is the external field, corresponding to a force (per square unit of the wall) $F = M_s H_0$.

The non-zero thickness δ of the domain wall has been completely ignored in the above analysis. This is justified as long as δ is much smaller that the diffusion length $\sqrt{D/\Omega}$.

3. Comparison with experiments

Detailed comparison with experiment is made difficult because no direct measurement of $\tau^{-1}(\sqrt{\Omega/D}, 0)$ is available. However, a detailed theory of the 'initial' susceptibility of ferrites has recently been proposed (Bouchaud and Zérah 1989, 1990a, b); this allows us to reach $\tau^{-1}(0, H_a^0)$, which is systematically found to be† $\tau^{-1} \simeq H_a^0 \simeq 10\text{–}10^2$ MHz. From the above discussion, one has

$$\mu \sim 2(a^2/\delta^2)\sqrt{J\tau}(1/\gamma 4\pi M_s)Ja. \tag{10}$$

For YIG (Teale 1980), one has $H_a^0 \simeq 100$ MHz (at room temperature). We take $J \simeq 5 \times 10^{13}$ Hz ($T_c \simeq 500$ K), $\delta = 100a$ ($a = 12.8$ Å) and $4\pi M_s \simeq 2000$ G. Choosing $\tau = 10^{-8}$ s, one finds that $\mu \simeq 4$ m s^{-1} Oe^{-1}, which is only one order of magnitude smaller than the value of 30 m s^{-1} Oe^{-1} determined by Teale (see below). If τ is a factor of 10 larger, the $\sqrt{\tau H_a^0}$ enhancement factor mentioned above leads to the correct order of magnitude for μ.

At this stage, we want to emphasize one point: experiments (Vella-Coleiro *et al* 1972, Teale 1980, Guyot *et al* 1988) show that the domain wall mobility is nearly composition independent and in particular is not related to the ferrimagnetic linewidth. For example, walls in pure YIG and ytterbium-doped YIG have the same mobility, while the linewidth is 100 times larger in the latter case. In order to reproduce this fact, the relaxation time at a small frequency $\tau(\sqrt{\Omega/D}, \Omega V/V^*)$ must not be dramatically dependent upon composition (in particular substitution) and should be quite different from its high-frequency counterpart. This would need a better experimental check than that based on the analysis of Bouchaud and Zérah (1989, 1990a, b).

3.1. *Domain of validity of the above estimate*

(i) The 'critical velocity' above which the mobility should start to depart from the linear regime is $V^* \sim 2\sqrt{DH_a^0}$, i.e. 140 m s^{-1}. This is consistent with the data of Teale (1980).

(ii) The diffusion length $l_d = \sqrt{D\tau}$ is of the order of $1000a$. It is thus justified to neglect the wall width compared with l_d. However, the assumption $l_d \ll \xi$ might not be correct. If this inequality is not satisfied, the above estimate for μ must be multiplied by a factor l_d/ξ. We are not aware of any estimate of ξ for the samples considered.

(iii) The characteristic frequency is $H_a^0(V/V^*)$, i.e. 100 MHz for $V = 140$ m s^{-1} (top of the linear regime). We are thus always in the 'low-frequency' regime as far as the estimate of τ is concerned; to probe frequencies such as 10 GHz (usual ferrimagnetic resonance experiments), one would need to go to velocities as high as 10^4 m s^{-1}. Hence our theory posssibly explains why the high-velocity experiments of Teale are not related to the ferrimagnetic resonance linewidth: the relevant frequency is not V/δ but $H_a^0(V/V^*)$.

(iv) We have been primarily concerned with *steady-state* determinations of the mobility, i.e. experiments done with a constant external field. If the wall is driven by an oscillating field $H\exp(i\omega t)$, equation (7) for the drag force immediately shows that the mobility is frequency independent for $\omega < \tau^{-1}$ and *grows* as $\sqrt{\omega}$ for higher frequencies.

(v) If the value of the intrinsic dissipation parameter α is sufficiently large, then the classical formula for the domain wall mobility should be valid, since $\mu^{-1} = \mu_{\rm intr}^{-1} + \mu_{\rm drag}^{-1}$. This occurs roughly when $\alpha > 0.01$.

4. Conclusion

We have thus proposed and analysed a mechanism of 'magnetic back-flow' for domain wall friction. It is based on a very general effect that a moving defect in a polarizable

† On depolarized samples of NiZn, MnZn and YIG ferrites.

medium creates an asymmetric polarization cloud around itself which leads, under mild assumptions about the intrinsic relaxation of the medium, to a drag force directly proportional to the velocity. For a magnetic domain wall, we argue that the polarizing field is due to the local transverse magnetization of the wall. Our treatment of this magnetostatic problem is not very elaborate and we only claim to obtain an *order of magnitude* of the wall's mobility. Then, an important physical statement (suggested by mobility experiments themselves) must be made concerning the intrinsic relaxation time of the medium. Traditionally, strictly following the Landau–Lifschitz phenomenological approach, this relaxation time is taken to be inversely proportional to the operating frequency, the proportionality constant being fixed by the ferrimagnetic linewidth. We take a completely different standpoint, corroborated by low-frequency experiments on demagnetized samples (Bouchaud and Zérah 1989, 1990a, b), as we think that this relaxation time does not diverge for small frequencies but instead is of the order of the frequency of the anisotropy field: the relaxation mechanism involved at high frequencies has no reason to be the same as the low-frequency one (Haas and Callen 1963).

The existence of these two very different relaxation times should, however, be directly confirmed experimentally; a more detailed comparison of our theory with experiments† needs an independent evaluation of τ, but also of the screening length ξ associated with the Bloch lines. Furthermore, the assumption of immobile Bloch lines could be criticized.

Finally, there is yet another unexplained fact revealed by experiments on clean YIG single crystals, which is the presence of a (very low) threshold field below which the wall does not move. The velocity appears to be non-zero just above threshold. One possibility could be the pinning of the domain wall to the dislocations (this could lead to interesting magnetomechanical effects). If this is the case, one expects, following Raphael and de Gennes (1989), a steep rise ($V \sim \sqrt{H - H_c}$) of the wall velocity in a small 'critical' region $\delta H/H_c \sim (a/l)^2$ before the linear regime is reached. (l is the distance between dislocations.)

Acknowledgments

JPB wishes to thank P Nozières for discussions. We also thank J Miltat and A Thiaville for drawing our attention to the work of Bar'yakthar.

References

Bar'yakthar V G 1984 *Zh. Eksp. Teor. Fiz.* **87** 1501
Bouchaud J P and Zérah P G 1989 *Phys. Rev. Lett.* **63** 1000
—— 1990a *J. Appl. Phys.* **67** 5512
—— 1990b to be published
de Gennes P G 1988 *C. R. Acad. Sci., Paris* **307** 1607
de Leeuw F H, Van den Doel R and Enz U 1980 *Rep. Prog. Phys.* **43** 689
Dorman V L and Sobolev V L 1988 *J. Phys. C: Solid State Phys.* **21** 5663
Eschenfelder K 1982 *Magnetic Bubble Technology* (Berlin: Springer)
Guyot M, Merceron T, Cagan V and Messekher A 1988 *Phys. Status Solidi* **106** 595
Haas C W and Callen H B 1963 *Magnetism* vol 1 ed G T Rado and H Suhl (New York: Academic)
Hagedorn F D and Gyorgy E M 1961 *J. Appl. Phys.* **32** 283

† In particular the temperature dependence of μ.

Harper H and Teale R W 1969 *J. Phys. C: Solid State Phys.* **2** 1926
Landau L and Lifschitz E M 1985 *Electromagnetism of Continuous Media* (Oxford: Pergamon)
Pines D and Nozières Ph 1966 *The Theory of Quantum Liquids* (New York: Benjamin)
Plefka T 1990 *Physica* A **162** 477
Raphael E and de Gennes P G 1989 *J. Chem. Phys.* **90** 7577
Teale R W 1980 *J. Phys. C: Solid State Phys.* **13** 2061
Thiele A A and Asselin P 1984 *J. Appl. Phys.* **55** 2584
Vella-Coleiro G P, Smith D H and Van Uitert L G 1972 *J. Appl. Phys.* **43** 2428
Williams H, Schokley W and Kittel C 1950 *Phys. Rev.* **80** 1090